

## *CPT*- and *B*-VIOLATION: THE $p - \bar{p}$ SECTOR \*

DALLAS C. KENNEDY

*Department of Physics, University of Florida,  
Gainesville FL 32611-8440 USA  
E-mail: kennedy@phys.ufl.edu*

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The *CPT* symmetry of relativistic quantum field theory requires the total lifetimes of particles and antiparticles be equal. Detection of  $\bar{p}$  lifetime shorter than  $\tau_p \gtrsim \mathcal{O}(10^{32})$  yr would signal breakdown of *CPT* invariance, in combination with *B*-violation. The best current limit on  $\tau_{\bar{p}}$ , inferred from cosmic ray measurements, is about one Myr, placing lower limits on *CPT*-violating scales that depend on the exact mechanism. Paths to *CPT* breakdown within and outside ordinary quantum mechanics are sketched. Many of the limiting *CPT*-violating scales in  $\bar{p}$  decay lie within the weak-to-Planck range.

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The *CPT* symmetry of local relativistic quantum field theory (LRQFT) has been tested in a number of elementary systems, in some cases to great accuracy.<sup>1,2,3</sup> *CPT* conservation rests on Lorentz invariance, locality, microcausality, and uniqueness of the vacuum.<sup>4</sup> With no obvious evidence to the contrary, we assume *CPT* symmetry to be good. But, as in the case of other discrete symmetries, the search for small, non-vanishing exceptions is important in its own right and as a check for residual forces outside the Standard Model.

*CPT* symmetry requires that the properties of matter and antimatter particles (charge, mass, magnetic moment) be related by charge conjugation. Less precise tests of *CPT* have also been conducted with other systems:<sup>5,6</sup> here the comparison of the proton and antiproton decay lifetimes,  $\tau_p$  and  $\tau_{\bar{p}}$ , is presented. The masses and electromagnetic properties of  $p$  and  $\bar{p}$  are still assumed identical for simplicity and charge ( $Q$ ) conservation unbroken. This test of *CPT* invariance is the first to be directly combined with baryon number ( $B$ ) violation.

### 1. Theoretical considerations

Such properties as intrinsic decay lifetimes cannot be limited with artificially produced antimatter at a level remotely approaching that possible with matter, as

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there is never enough antimatter available for a long enough time. As a result, the decay lifetime of single antiprotons has been limited by laboratory measurements to no more than  $\tau_{\bar{p}} > 3.4$  months<sup>7</sup>; while the decay of stored accelerator  $\bar{p}$ 's is somewhat more restricted,  $\tau_{\bar{p}}/B(\bar{p} \rightarrow \mu^- \gamma) > 0.05 \times 10^6$  yr (0.05 Myr) for the most sensitive exclusive mode.<sup>8</sup>

With timescales much longer than possible in the laboratory, astrophysical antimatter processes provide some compensation for antimatter measurements. The resulting antimatter abundances can be inferred from cosmic ray measurements. The intrinsic timescale of cosmic antiprotons, produced by secondary processes and stored temporarily in our Galaxy, is approximately 10 Myr.<sup>9,10</sup>

Since  $\tau_p \gtrsim \mathcal{O}(10^{32})$  yr,<sup>6</sup> a detected  $\tau_{\bar{p}} \lesssim 10^7$  yr would be *prima facie* evidence of *CPT* violation, as otherwise  $\Gamma_{\bar{p}} = \Gamma_p$  automatically. A relevant cosmic ray limit has been recently derived:<sup>11</sup>  $\tau_{\bar{p}} > 0.8$  Myr (90% C.L.).

### 1.1. *CPT violation in standard quantum mechanics*

Without modifying basic quantum dynamics, *CPT*-violating effects can be introduced as small QFT modifications to the Standard Model (SM).<sup>12</sup> Such extensions can violate one or more of any of the four preconditions for *CPT* invariance, but the easiest to implement is Lorentz violation.<sup>13</sup>

If we assume that only one new, large scale,  $M_X$ , is involved in the *CPT* violation, then the details of the interactions can be bypassed with naive dimensional analysis. On dimensional grounds, a QFT operator in an extended SM Lagrangian density with canonical (mass) dimension  $n > 4$  must be suppressed by  $M_X^{4-n}$ . The resulting decay rate must be of the form

$$\Gamma_{\bar{p}} \sim m_p \cdot [m_p/M_X]^{2n-8} . \quad (1)$$

Ignoring dimensionless factors and neglecting  $\Gamma_p$  compared to  $\Gamma_{\bar{p}}$ , the new *CPT*-violating scale  $M_X$  is then related to the  $\bar{p}$  lifetime by  $M_X = m_p \cdot [m_p \tau_{\bar{p}}]^{1/(2n-8)}$ . Current limits on  $\tau_{\bar{p}}$  result in scales  $M_X \lesssim M_{\text{Planck}}$  (see section below).

### 1.2. *CPT violation via extensions of quantum mechanics*

An additional path to *CPT* violation appears if we modify standard quantum mechanics. The most general description of a mixed quantum state is given by the density matrix  $\hat{\rho}(t)$  evolving via the Liouville-von Neumann (LvN) equation:

$$i \frac{d\hat{\rho}(t)}{dt} = [\hat{\mathbf{H}}, \hat{\rho}] , \quad (2)$$

where the Hamiltonian  $\hat{\mathbf{H}}$  is usually assumed Hermitian. Unitarity requires  $\text{Tr}(\hat{\rho}) = 1$ , and equation (2) preserves that condition. Irreversible decay can be incorporated by dropping the hermiticity of  $\hat{\mathbf{H}}$  and writing  $\hat{\mathbf{H}} = \hat{\mathbf{M}} - i\hat{\mathbf{\Gamma}}/2$ , where  $\hat{\mathbf{M}}$  and  $\hat{\mathbf{\Gamma}}$  are Hermitian and positive semi-definite. Unitarity is no longer preserved by the LvN

equation, and  $\text{Tr}(\hat{\rho})$  decreases in time. Alternatively, the *entropy*  $S(t)$  increases:  $\dot{S} = -\text{Tr}(\hat{\rho} \ln \hat{\rho})$ ,  $\dot{S} \geq 0$ .

Inspired by his discovery of quantum black hole radiation, Hawking proposed to extend quantum mechanics<sup>14</sup> with new terms in (2) that transform pure to mixed states. Such a non-standard quantum mechanics (NSQM) was later shown by Page to imply *CPT* violation and by Banks, Susskind, and Peskin to create a conflict between locality and Lorentz invariance (energy-momentum conservation).<sup>15</sup> The additional terms in question add to other possible *CPT*-violating effects that can already occur in ordinary quantum field theory. This NSQM formalism has been applied to neutral kaons<sup>16</sup> and solar neutrinos.<sup>17</sup>

A similar approach can be taken with the  $p - \bar{p}$  system, working in a two-dimensional space,  $(p, \bar{p})$ , with a density matrix  $\hat{\rho}$ . Decompose  $\hat{\rho}$  in a general basis:  $\hat{\rho} = \rho^0 \hat{\mathbf{1}} + \rho^i \hat{\sigma}^i$ , and  $\hat{\mathbf{M}}$  and  $\hat{\mathbf{\Gamma}}$  similarly, where  $\hat{\mathbf{1}}$  is the identity and  $\hat{\sigma}^i$  the Pauli matrices. Take the most general linear extension of the LvN equation in this space:

$$\frac{d\hat{\rho}}{dt} = 2\epsilon^{ijk} M^i \rho^j \hat{\sigma}^k - \Gamma^0 \hat{\rho} - \Gamma^i (\rho^0 \hat{\sigma}^i + \rho^i \hat{\mathbf{1}}) - h^{0j} \rho^j \hat{\mathbf{1}} - h^{j0} \hat{\sigma}^j - h^{ij} \hat{\sigma}^i \rho^j, \quad (3)$$

with the summation convention assumed and the last three terms as the NSQM extensions.

The equation (3) can be simplified with reasonable assumptions: absorbing as many terms as possible into shifts of  $\hat{\mathbf{M}}$  and  $\hat{\mathbf{\Gamma}}$ ; equality of masses,  $m_p = m_{\bar{p}}$ ; and requiring  $\dot{S} \geq 0$  (positivity). These requirements eliminate  $h^{0j}$  and  $h^{j0}$  and imply  $h^{ij} \geq 0$ .<sup>16</sup> We also assume  $\Delta Q = 0$ , which further simplifies (3) by forcing all off-diagonal terms to zero and leaving only  $h \equiv h^{33}$  (not to be confused with Planck's constant). The only other *CPT*-violating effect arises from  $\Delta\Gamma \equiv \Gamma_{\bar{p}} - \Gamma_p$ , the component of  $\hat{\mathbf{\Gamma}}$  proportional to  $\hat{\sigma}^3$ . The density matrix  $\hat{\rho}$  has only diagonal components  $\rho_+$  and  $\rho_-$ , and (3) can be reduced to two coupled equations:

$$\dot{\rho}_{\pm}(t) = -[\bar{\Gamma} \mp \Delta\Gamma/2 + h/2] \cdot \rho_{\pm} + h\rho_{\mp}/2, \quad (4)$$

with  $\bar{\Gamma} \equiv (\Gamma_{\bar{p}} + \Gamma_p)/2$ . Note that baryon number  $B$  is the operator  $\hat{\mathbf{B}} = \hat{\sigma}^3$ , and its expectation value is

$$\langle B(t) \rangle = \text{Tr}[\hat{\mathbf{B}}\hat{\rho}(t)] . \quad (5)$$

Subject (4) to eigenmode analysis and take  $\text{Tr}[\rho(0)] = 1$ . The extensions of (3) imply nothing about locality, and these terms in general are nonlocal and/or acausal. So assume an initial state of matter consistent with typical laboratory conditions, but also with the presumed state of the Universe:  $\rho_+(0) = 1 - \varepsilon$ ,  $\rho_-(0) = \varepsilon$ , with  $\varepsilon \rightarrow 0$ . The effective decay rates, if small, are then

$$\begin{aligned} \Gamma_p^{\text{eff}} &= -\dot{\rho}_+(0) = (1 - \varepsilon)\Gamma_p + h/2 \rightarrow \Gamma_p + h/2, \\ \Gamma_{\bar{p}}^{\text{eff}} &= -\dot{\rho}_-(0) = \varepsilon\Gamma_{\bar{p}} + (1 + 2\varepsilon) \cdot h/2 \rightarrow h/2. \end{aligned} \quad (6)$$

Table 1: *CPT*- and *B*-violating scale limits from  $p$  lifetime  $\tau_p = 10^{32}$  yr and  $\bar{p}$  lifetime  $\tau_{\bar{p}} = 10^7$  yr (see text).

$n$	$M_X$ (GeV)	$k$	$M_Y$ (GeV)
5	$2 \times 10^{19}$	1	$9 \times 10^{63}$
6	$4 \times 10^9$	2	$9 \times 10^{31}$
7	$3 \times 10^6$	3	$2 \times 10^{21}$
8	$6 \times 10^4$	4	$9 \times 10^{15}$
9	$7 \times 10^3$	5	$6 \times 10^{12}$
10	$2 \times 10^3$	6	$4 \times 10^{10}$

Note that the NSQM evolution corrections are partly independent of the  $\bar{p}$  concentration  $\varepsilon$ . The *relative*  $\bar{p}$  decay rate  $-\dot{\rho}_-(0)/\varepsilon$  is enhanced by the small  $\bar{p}$  concentration in astrophysical situations: <sup>9,10,11</sup>  $\varepsilon^{-1} \sim 10^{4-5}$ .

Again assume only one new, large *CPT*-violating scale,  $M_Y$ , that gives rise to  $h$  in this *B*-violating sector; naive dimensional analysis again indicates  $h = m_p^{k+1}/M_Y^k$ ,  $k \geq 1$ , ignoring dimensionless factors. Then  $M_Y = m_p \cdot [m_p/h]^{1/k}$ . For higher-dimensional (larger  $k$ ) cases in the following section, the limiting scales again fall in the weak-to-Planck range.

## 2. Implications for *CPT* violation and associated scales

Approximate limits on  $M_X$ ,  $h$ , and  $M_Y$  can be inferred from the lifetime limits  $\tau_p \gtrsim 10^{32}$  yr and  $\tau_{\bar{p}} \gtrsim 10^7$  yr. <sup>6,11</sup> The lower limits on  $M_X$  are listed in the left two columns of Table 1. Note that the *largest*  $M_X$  is about  $M_{\text{Planck}}$ , while the next largest lies within the “intermediate” range associated with left-right unification and Peccei-Quinn symmetry breaking. <sup>18</sup> From higher-dimensional operators come lower scales close to the weak scale. These may seem implausible, but might fit into “large-radius”  $D \geq 4$ -brane compactifications with macro- or mesoscopic modifications to gravity and TeV-scale unification. <sup>19</sup>

Now consider the measured *B*-violation limits on NSQM extensions. Here it is  $\tau_p$ , not  $\tau_{\bar{p}}$ , that restricts  $h$ , as  $\tau_p$  is so much better constrained:  $h \lesssim 10^{-55}$  eV! But the two right columns of Table 1 show that considering the NSQM term  $h$  in terms of naive dimensional analysis results, for higher-dimensional operators,  $k \geq 4$ , in scales  $M_Y$  lower than  $M_{\text{Planck}}$  and hence not so implausible. The  $k = 4$  case falls close to the standard grand unification scale; the  $k = 5$  and 6 cases are in the “intermediate” range again.

## 3. Summary and Outlook

*CPT* violation, in the context of the *B*-violating  $p$  and  $\bar{p}$  decays, can be limited with the best current limits on  $\tau_p$  (laboratory) and  $\tau_{\bar{p}}$  (cosmic rays), providing a test of *CPT* invariance different from non-baryonic limits. Assuming one new scale  $M_X$  or  $M_Y$  associated with the *CPT*- and *B*-violating interactions, these limits imply lower bounds on  $M_X$  or  $M_Y$  that may, depending on the exact mechanism,

fall in the usual weak-to-Planck range.

Improvements in laboratory and cosmic ray antiproton measurements<sup>20</sup> will further probe the combination of  $CPT$  and baryon number violation. Unified string or brane theories, while preserving  $CPT$  at the dynamical level, could break it spontaneously upon compactification,<sup>21</sup> while the full effect of quantum gravity on fundamental symmetries remains an enigma. Perhaps a symmetry such as  $CPT$  today so obvious may fall, like parity and  $CP$ , under the ax of an unexpected experimental result.

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